Occupation Mobility, Human Capital and the Aggregate Consequences of Task-Biased Innovations[1]

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November 2019

[1] The views expressed here are those of the authors and do not necessarily reflect the opinion of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

Introduction

- **Task-Biased Technological & Organizational Innovations:**
  - Displace workers from tasks.
  - Biased against some occupations & workers.
  - **Literature:** computers, robots; automation, off-shoring.
    - polarization, inequality.
    - factor shares.
  - **Key Margins:**
    - How easy/difficult is to substitute factors for different tasks.
    - How easy/difficult can workers switch tasks/labor markets.
    - Dynamic investments and reallocation decisions.

This Paper: What We Do

- Develop and solve a **dynamic** Roy model:
  - Occupation mobility & human capital accumulation.
  - Aggregation. Individual and Aggregate dynamics.
- Develop and solve a **production assignment** model of workers to tasks:
  - Occupation specific 'races' of man vs. machine.
  - Aggregate production function.
- Consider a **dynamic general equilibrium** setting:
  - Simple demographics to distinguish: invidividuals, cohorts and aggregate.
  - Solve for BGP & transition dynamics.
- **Quantitative Analysis:** U.S. Task-biased innovations:1980-2010.
  - **Accounting** for:
    - Employment/Capital Income shares.
    - Income Inequality.
    - Aggregate Income.
  - **Counterfactuals:** Occupational mobility.

This Paper: What We Show

- **A recursive Roy model:** E.V. shocks & CRRA preferences:
  - Sharp solutions: values & transitions (occupation mobility.)
  - Aggregation & Ergodicity of Workers Across Occupations.
  - **Human Capital:**
    - Individual growth dynamics.
    - Aggregation: Levels vs. Growth.
  - **Aggregate Impact of Task-Biased Innovations:**
    - Long Transitions.
    - Large Cross-BGP differences.
- **Equilibrium assignment of workers/machines to tasks:**
  - Aggregate production function: Nested CES.
  - Occupation Specific E. of Substitution Workers-Machines.
  - Endogenous Labor Share of Output.

This Paper: What We Show (cont'd)

- In our **dynamic general equilibrium** setting:
  - BGP: Generic Existence.
  - Transition dynamics: dynamic hat-algebra.
- **Quantitative Analysis:** U.S. Labor Markets
  - **Accounting:** Task-biased innovations:1980-2010 can account for:
    - Observed Changes in Employment/Capital Income shares.
    - Substantial Increase in Income Inequality.
    - 75% Higher Aggregate Output.
  - **Counterfactuals:** Lower Occupation Mobility.
    - More Inequality.
    - Less Employment Polarization.
    - Lower Aggregate Income.

Contributions to Related Literature

- **Labor Market Polarization: e.g.:** Autor et al. (2003), Goos and Manning (2007), Acemoglu and Autor (2011), Acemoglu and Restrepo (2019).
  - Quantitative, dynamic GE. Occupation heterogeneity subst. man-machines.
  - Forward-looking workers with job mobility and endogenous human capital.
- **Capital-Skill Complementarity/Skill-Biased Technical Change: e.g.:** Krusell et al. (2000), Greenwood et. al (1997)
  - Occupations and multiple skills with mobility; endogenous assignment.
- **Quantitative GE Roy Models: e.g.:** Burstein et al. (2019), Hsieh et al (2019), Lagakos and Waugh (2013), Galle et al. (2017)
  - Recursive framework, CRRA preferences. Initial and Transition probabilities.
- **Dynamic Roy Models & Human Capital: e.g.:** Lee and Wolpin (2006), Adao et al. (2018)
  - GE with many occupations. Human capital: initial and transition choices .
- **Cross-Country Differences in Life-Cycle Earnings Profiles: e.g.:** Lagakos et al. (2018),
  - Occupations choices; Implications of task-biased technology.
- **Aggregation and Factor Shares: e.g.:** Karabarbounis and Neiman (2013); Martinez (2019)
  - Assignment; heterogeneous occupations wrt substitution workers/machines.

Road Map

1. **U.S. Evidence:** Task-Biased Innovations and Occupations.
2. **A Dynamic Roy Model:**
   - Individual Problems.
   - Aggregation.
   - Numerical Illustration.
3. **Production:** Assignment of Tasks to Workers/Machines.
   - Characterization.
   - Aggregation.
4. **General Equilibrium:**
   - BGP: Existence.
   - Transition Dynamics: dynamic hat algebra.
5. **Quantitative Analysis.**
6. **Conclusions.**
7. *If time permits:* **Extensions and Variations.**

U.S. Evidence on Task-Biased Innovations and Occupations.

Some Evidence on Task-Biased Technical Change

- Use a cross-section of US commuting zones (CZ) indexed by i:

$$\Delta\left(\frac{E_i^{\text{occ } j}}{Pop_i}\right) = \alpha + \beta \cdot \Delta\mathbf{PCs}_i + \delta \cdot \Delta\mathbf{Robots}_i + \gamma X_i + \varepsilon_i.$$

- Dependent variable: $\Delta\left(\frac{E_i^{\text{occ } j}}{Pop_i}\right)$ for broad occupations j:
  - $j = \{\texttt{routine, non-routine}\} \times \{\texttt{manual, cognitive}\}$
- Impact on employment to the CZ exposure to:
  - Computers (PCs)
  - Automation (Robots)
- Follow Autor & Dorn (2013) and Acemoglu & Restrepo (2019).
  - $\mathbf{PCs}_i$, $\mathbf{Robots}_i$ instrumented as in those papers.

U.S. Evidence on Task-Biased Technical Change

	Non-Routine Cognitive			Non-Routine Manual		
	(1)	(2)	(3)	(1)	(2)	(3)
Change in PC's per worker	1.163**		1.142*	0.596		0.585
	(2.59)		(2.55)	(1.69)		(1.71)
Adjusted Exposure to Robots		-0.087	-0.082		-0.047	-0.044
		(-1.61)	(-1.55)		(-0.66)	(-0.66)
observations	660	722	660	660	722	660
Adjusted R^2	0.172	0.164	0.175	0.174	0.170	0.176

	Routine Cognitive			Routine Manual		
	(1)	(2)	(3)	(1)	(2)	(3)
Change in PC's per worker	-3.158***		-3.132***	1.471*		1.314*
	(-6.22)		(-6.25)	(2.28)		(2.19)
Adjusted Exposure to Robots		0.118	0.105		-0.622***	-0.616***
		(0.93)	(1.01)		(-6.04)	(-6.45)
observations	660	722	660	660	722	660
Adjusted R^2	0.170	0.095	0.174	0.283	0.361	0.364

A Dynamic Roy Model

The Worker's Problem

- **Workers' Preferences:** discount factor: $0 < \beta < 1$; CRRA $\gamma \geq 0$.

$$U_t = \frac{[c_t]^{1-\gamma}}{1-\gamma} + E\left[\sum_{s=1}^{\infty} \beta^s \frac{[c_{t+s}]^{1-\gamma}}{1-\gamma}\right],$$

- **Occupations,** $J \geq 2$ **: Wage vector** $w \in R^J$, given, fixed over time.
  - Idiosyncratic stochastic sequences $\{\epsilon_t\}_{t=0}^{\infty}$,

$$\epsilon_t = \left[\epsilon_t^1,\, \epsilon_t^2,\, ...\epsilon_t^{\ell}...,\epsilon_t^J\right] \in \mathbb{R}_+^J$$

- **Human Capital:** Initial level $h > 0$:
  - *Current earnings/consumption:* $c_t = w_j h$.
  - *Transferability Matrix* (across occupations, next period): $\boldsymbol{\tau} : J \times J \to \mathbb{R}_+$.

$$h_t\, \tau\, [j, \cdot] \otimes \epsilon_t \in \mathbb{R}_+^J,$$

  $\otimes$: element-by-element product.

  - *Ex-post evolution*

$$h_{t+1} = h_t \tau \left[j_t,\, \ell_{t+1}\right] \epsilon_{\ell,t}.$$

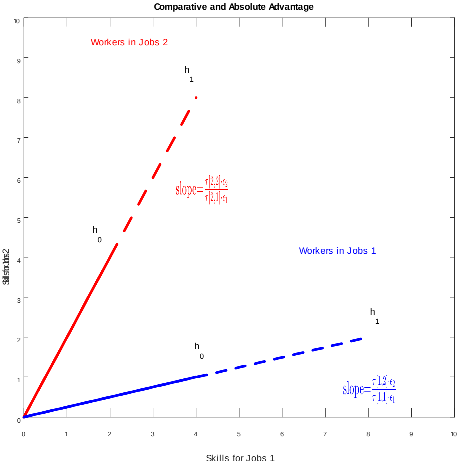
Absolute and Comparative Advantage

- **Absolute advantage: Magnitude** of h
  - **Endogenous accumulation**:

$$h_t = h_0 \prod_{s=0}^{t-1} \tau \left[j_s, \ell_{s+1} \right] \epsilon_{\ell_{s+1}}$$

- **Comparative advantage: Direction** of h
  - **Markovian:** Current (j, ϵ):

$$\frac{\tau \left[j, \ell \right]}{\tau \left[j, j \right]} \frac{\epsilon_\ell^t}{\epsilon_j^t}.$$



The Worker's Problem

- **Bellman Equation (BE):**

$$V(j,h,\epsilon) = \frac{\left(w^j\, h\right)^{1-\gamma}}{1-\gamma} + \beta \max_{\ell} \left\{ E_{\epsilon'} V\left[\ell,\ h',\ \epsilon'\right] \right\},$$

  - $E_{\epsilon'}[\cdot]$ is the expectation over the next period's ϵ'
  - $h' = h\,\tau_{j\ell}\epsilon_{\ell}$.

- **Factorizaton:** $V(j,h,\epsilon) = v(j,\epsilon)\, h^{1-\gamma}$

  - **Homogeneity (1) Objective function; (2) Feasible sets.**

$$v(j,\epsilon)\, h^{1-\gamma} = \left(\frac{\left(w^j\right)^{1-\gamma}}{1-\gamma} + \beta \max_{\ell} \left\{ E_{\epsilon'}\left[v\left(\ell,\epsilon'\right)\right] \left(\tau_{j,\ell}\epsilon^{\ell}\right)^{1-\gamma} \right\} \right) h^{1-\gamma}.$$

  - **Normalized BE**:

$$v(j,\epsilon) = \frac{\left(w^j\right)^{1-\gamma}}{1-\gamma} + \beta \max_{\ell} \left\{ \left(\tau_{j,\ell}\epsilon^{\ell}\right)^{1-\gamma} E_{\epsilon'}\left[v\left(\ell,\epsilon'\right)\right] \right\},$$

The Worker's Problem

- **Conditional Expectations:**.

$$v^j \equiv E_\epsilon \left[v\left(j,\epsilon\right)\right].$$

- **Recursion:**

$$\mathbf{v^j} = \begin{cases} \frac{\left(w^j\right)^{1-\gamma}}{1-\gamma} + \beta E_\epsilon \max_\ell \left[\left\{ \left[\tau_{j,\ell}\, \epsilon^\ell \right]^{1-\gamma} \mathbf{v}^\ell \right\} \right], & \text{for } \gamma \neq 1, \\ \ln w^j + \beta E_\epsilon \left[\max_\ell \left\{ \mathbf{v}^\ell + \frac{\ln\left(\tau_{j,\ell}\epsilon^\ell\right)}{1-\beta} \right\} \right], & \text{for } \gamma = 1. \end{cases}$$

- **Random realized values:**

$$v\left(j,\epsilon\right) = \frac{\left(w^j\right)^{1-\gamma}}{1-\gamma} + \beta \max_\ell \left\{ \left(\tau_{j,\ell}\epsilon^\ell\right)^{1-\gamma} \mathbf{v^j} \right\}.$$

The Worker's Problem: Generic

- **Assumptions:**
  - $\tau_{j,\ell} > 0$ for all j, ℓ.
  - support of ϵ^{ℓ} is $[0, \infty)$ for all ℓ.
- **Growth Factors:** Let $\bar{\Phi} \equiv \max\{\Phi_j\}$, where:

$$\Phi_j \equiv \begin{cases} E_\epsilon \max_\ell \left\{ \left[\tau_{j,\ell}\, \epsilon_\ell\right]^{1-\gamma} \right\}, & \text{for } 0 \leq \gamma < 1, \\ E_\epsilon \max_\ell \left\{ \ln\left(\tau_{j,\ell}\epsilon_\ell\right) \right\}, & \text{for } \gamma = 1. \\ E_\epsilon \min_\ell \left\{ \left[\tau_{j,\ell}\, \epsilon_\ell\right]^{1-\gamma} \right\}, & \text{for } \gamma > 1. \end{cases}$$

Lemma. There exists a unique, finite $v \in \mathbb{R}^J$ that solves the BE:

(a) For all $0 < \gamma \neq 1$, if $\beta\bar{\Phi} < 1$,

(b) For $\gamma = 1$, if $\beta < 1$ and $-\infty < \Phi_j < +\infty$, $\forall j$.

The Worker's Problem: E.V. Distributions

Lemma. **Derived Distributions.** Let ϵ^{ℓ} ~Frechet(α, λ_{ℓ}): $F_{\epsilon}(\epsilon) = e^{-\left(\frac{\epsilon}{\lambda_{\ell}}\right)^{-\alpha}}$.

$$\text{Define:} \qquad x_{\ell} \equiv \begin{cases} \left(\epsilon^{\ell}\right)^{1-\gamma} & \text{for } 0 \leq \gamma \neq 1 \\ \ln\left(\epsilon^{\ell}\right) & \text{for } \gamma = 1. \end{cases}$$

Then:

$$x_{\ell} \sim \begin{cases} \text{Frechet}\left(\frac{\alpha}{1-\gamma}, (\lambda_{\ell})^{1-\gamma}\right) & \text{for } 0 \leq \gamma < 1, \\ \text{Gumbel}\left(\frac{1}{\alpha}, \ln(\lambda_{\ell})\right) & \text{for } \gamma = 1, \\ \text{Weibull}\left(\frac{\alpha}{\gamma-1}, (\lambda_{\ell})^{\gamma-1}\right) & \text{for } \gamma > 1. \end{cases}$$

Frechet shocks & CRRA Pref. $\Longrightarrow$ **E.V. distributed Continuation Values.**

The Worker's Problem: E.V. Distributions

Theorem. **Individual Problems.** Let $\epsilon_\ell \sim$ Frechet$(\lambda_\ell,\, \alpha)$, $w \in \mathbb{R}^J_{++}$,cond. Lemma 1:

(i) Fixed points: $\exists!\; v \in \mathbb{R}^J$ that solve the recursions

$$\mathbf{v^j} = \begin{cases} \frac{\left(w^j\right)^{1-\gamma}}{1-\gamma} + \beta\,\Gamma\left(1 - \frac{1-\gamma}{\alpha}\right)\left[\sum_{k=1}^{J} \left(\mathbf{v^k}\right)^{\frac{\alpha}{1-\gamma}} \left(\tau_{jk}\lambda_k\right)^{\alpha}\right]^{\frac{1-\gamma}{\alpha}}, & \text{if } \gamma \in [0,1) \\ \log\left\{ w^j \left[\sum_{k=1}^{J} \exp\left(\alpha\, \left[(1-\beta)\mathbf{v^k} + \log\left(\tau_{j\ell}\lambda_\ell\right) + \kappa\right]\right)\right]^{\frac{\beta}{\alpha\,(1-\beta)}} \right\}, & \text{if } \gamma = 1. \\ \frac{\left(w^j\right)^{1-\gamma}}{1-\gamma} - \beta\,\Gamma\left(1 - \frac{1-\gamma}{\alpha}\right)\left[\sum_{k=1}^{J} \left(-\mathbf{v^k}\right)^{\frac{\alpha}{1-\gamma}} \left(\tau_{jk}\lambda_k\right)^{\alpha}\right]^{\frac{1-\gamma}{\alpha}}, & \text{if } \gamma > 1. \end{cases}.$$

The Worker's Problem: E.V. Distributions

Theorem. Individual Problems. Let $\epsilon_\ell \sim$ Frechet(λ_ℓ, α), $w \in \mathbb{R}^J_{++}$,cond. Lemma 1:

(ii) Transitions: $\exists!$ optimal policy functions, moving from j to ℓ

$$\mu(j,\ell) = \begin{cases} \dfrac{\left[\lambda_\ell \tau_{j\ell}\left(\mathbf{v}^\ell\right)^{\frac{1}{1-\gamma}}\right]^\alpha}{\sum_{k=1}^{J}\left[\lambda_k \tau_{jk}\left(\mathbf{v}^{\mathbf{k}}\right)^{\frac{1}{1-\gamma}}\right]^\alpha}. & \text{if } \gamma \in [0,1) \\ \dfrac{\exp\left(\alpha\,(1-\beta)\mathbf{v}^\ell + \alpha\log(\tau_{j\ell}) + \alpha\log(\lambda_\ell)\right)}{\sum_{k=1}^{J}\exp\left(\alpha\,(1-\beta)\mathbf{v}^{\mathbf{k}} + \alpha\log(\tau_{jk}) + \alpha\log(\lambda_k)\right)} & \text{if } \gamma = 1. \\ \dfrac{\left[\lambda_\ell \tau_{j\ell}\left(-\mathbf{v}^\ell\right)^{\frac{1}{1-\gamma}}\right]^\alpha}{\sum_{k=1}^{J}\left[\lambda_k \tau_{jk}\left(-\mathbf{v}^{\mathbf{k}}\right)^{\frac{1}{1-\gamma}}\right]^\alpha}. & \text{if } \gamma > 1. \end{cases} .$$

Aggregate Implications: Distribution of Workers

- **Workers Across Occupations:**
  - Let $\theta_t = \left[\theta_t^1, ..., \theta_t^J\right]$ be the fraction of workers in the J occupations:

$$\theta_{t+1} = \mu \, \theta_t .$$

  - **Mixing Condition:** $\lambda_\ell, \, \tau_{j,\ell} > 0, \implies \mu\,(j, \ell) > 0$ for all j, ℓ.
    - **Existence, Uniqueness** of an invariant distribution:

$$\theta_\infty = \mu \, \theta_\infty ,$$

    - **Convergence:** $\{\theta_t\}_{t=0}^{\infty} \to \theta_\infty$ from any initial distribution θ_0.

Human Capital: Aggregation and Distribution Dynamics

- **Human Capital Across Occupations:**
  - $\phi_t^j(\cdot)$: measure, human capital levels, workers in j, period t. Then:

$$H_t^j = \theta_t^j \int_0^\infty h\phi_t^j(dh),$$

Lemma. For all $0 \leq \gamma \neq 1$, the expectation of ϵ_ℓ of workers switching from j to ℓ:

$$E\left[\epsilon_\ell | \Omega_{j\ell}\,\epsilon_\ell^{1-\gamma} = \max_k \left\{\Omega_{j\ell}\epsilon_k^{1-\gamma}\right\}\right] = \Gamma\left(1 - \frac{1}{\alpha}\right)\lambda_\ell \left[\mu\left(j,\ell\right)\right]^{-\frac{1}{\alpha}}.$$

- Define matrix $\mathcal{M}$:

$$\mathcal{M}\left(j,\ell\right) = \Gamma\left(1 - \frac{1}{\alpha}\right)\tau_{j\ell}\,\lambda_\ell \left[\boldsymbol{\mu}\left(j,\ell\right)\right]^{1-\frac{1}{\alpha}}.$$

Human Capital: Aggregation and Distribution Dynamics

- $H_t = \left[H_t^1 \ldots H_t^J\right]$: aggregate human capital across occupations:

$$H_{t+1} = H_t \mathcal{M}.$$

  - **No invariant distribution:** $\mathcal{M}$ not an stochastic matrix.

- **Positivity:** $\lambda_\ell, \tau_{j,\ell}, \mu_{j,\ell} > 0 \implies \mathcal{M}_{j,\ell} > 0$: *Perron-Frobenius Theorem:.*
  - largest eigenvalue of $\mathcal{M}$: multiplicity one, real and positive.
  - associated eigenvector: strictly positive coordinates.
  - Asymptotics from any initial distribution H_0:

$$H_{t+1}^j = G_H \times H_t^j, \text{ for all } j.$$

- Endogenous Growth: G_H root depends on μ.
  - **Once-and-for-all Δw $\implies$ permanent $\Delta \mathbf{G}_H$.**

Cohorts

Perpetual Youth: Death/Entry Rate $0 < \delta < 1$.

- **Active (old) workers:** Same as before, but $\beta = \beta_{\text{pure}} (1 - \delta)$.
- **New Workers:** All with $h_{-1} = 1$. Vector $\tau^0 = \left[\tau^0 (1) \ldots \tau^0 (J)\right]$.

Initial Employment:

$$\theta^0 (j) = \frac{\left[\lambda_j \tau^{0,j} \left(-\mathbf{v}^{\mathbf{j}}\right)^{\frac{1}{1-\gamma}}\right]^{\alpha}}{\sum_{k=1}^{J} \left[\lambda_k \tau^{0,k} \left(-\mathbf{v}^{\mathbf{k}}\right)^{\frac{1}{1-\gamma}}\right]^{\alpha}},$$

Initial Human Capital:

$$H^0 (j) = \Gamma \left(1 - \frac{1}{\alpha}\right) \tau^{0,j} \lambda_j \left[\theta^0 (j)\right]^{1-\frac{1}{\alpha}} .$$

Cohorts: Aggregates

- **Time evolution:**
  - *Employment:.*

$$\begin{aligned}\theta_{t+1} &= \delta\theta^0 + (1-\delta)\,\theta_t\mu \\ &= \delta\theta^0 \sum_{\tau=0}^{t} \left[(1-\delta)\,\mu\right]^\tau .\end{aligned}$$

  - *Human Capital:.*

$$\begin{aligned}H_{t+1} &= \delta H^0 + (1-\delta)\,H_t\mathcal{M} \\ &= \delta H^0 \sum_{\tau=0}^{t} \left[(1-\delta)\,\mathcal{M}\right]^\tau .\end{aligned}$$

- **Steady States?**
  - *Employment:.* **always.**

$$\lim_{t\to\infty} \theta_{t+1} = \delta\theta^0 \left[I - (1-\delta)\,\mu\right]^{-1} .$$

  - *Human Capital:.* **iff** $G_H < (1-\delta)^{-1}$:

$$\lim_{t\to\infty} H_{t+1} = \delta H^0 \left[I - (1-\delta)\,\mathcal{M}\right]^{-1} ;$$

  o/w H_t grows over time.

Illustrating the Impact of Task-Biased Technical Change

Four Occupations: $J = 4$. {Manual, Cognitive} $\times$ {Routine, Non-Routine}

- **Unitary wages, Human Capital Transferability**

$$\mathbf{w} = \begin{bmatrix} w_{\mathbf{MR}} \\ w_{\mathbf{MN}} \\ w_{\mathbf{CR}} \\ w_{\mathbf{CN}} \end{bmatrix} = \begin{bmatrix} 1.25/\mathbf{tbtc} \\ 1 \\ 1.25/\mathbf{tbtc} \\ 1 \end{bmatrix}; \quad \boldsymbol{\tau}_{[j,\ell]} = \begin{bmatrix} \mathbf{1.00} & \tau_{RN} & \tau_{MC} & \tau_{MC}\tau_{RN} \\ \tau_{NR} & \mathbf{1.015} & \tau_{MC}\tau_{NR} & \tau_{MC} \\ \tau_{CM} & \tau_{CM}\tau_{RN} & \mathbf{1.015} & \tau_{RN} \\ \tau_{CM}\tau_{NR} & \tau_{CM} & \tau_{NR} & \mathbf{1.05} \end{bmatrix}.$$

with:

- $\tau_{CM} = 0.95$; $\tau_{NR} = 0.95$; $\tau_{MC} = 0.7$; $\tau_{RN} = 0.6$.
- $\lambda = [1, 1, 1]' / \Gamma(1 - 1/\alpha)$.

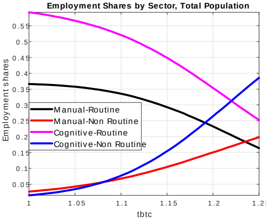
- **Flow of new workers:** $\delta = 0.04$. **Initial** τ^0:

$$\tau^0 = [1, \ 1, \ 1, 0.65];$$

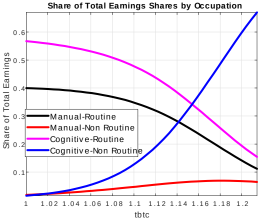
tbtc= *"task-biased technical change"* $\in [1, \ 1.25 \ ]$.

Cross-Steady States/BGPs

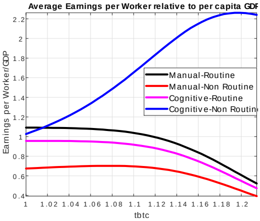
- BGPs for **tbtc="task-biased technical change"** $\in [1, 1.25]$.



Employment Shares



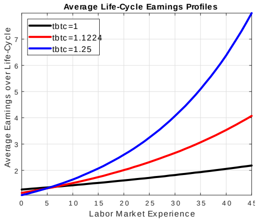
Output Shares



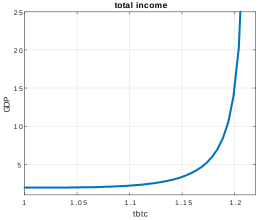
Avg. earnings/GDP

Cross-Steady States/BGPs

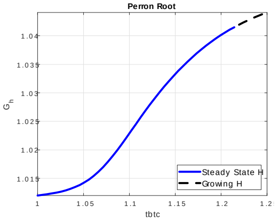
- BGPs for **tbtc=“task-biased technical change"** $\in [1, 1.25]$.



Life-cycle profiles



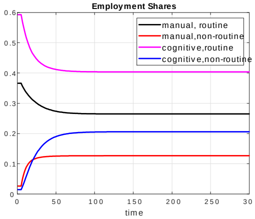
Output per worker



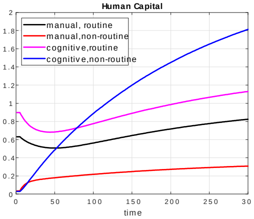
Perron Root of $\mathcal{M}$

Transition from one BGP to another

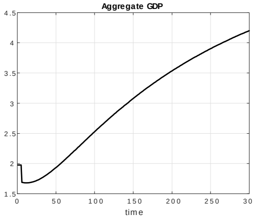
- unanticipated, permanent, once-and-for-all at $t = 6$:
  - from $tbtc = 1$ to $tbtc = 1.5$.



Employment shares



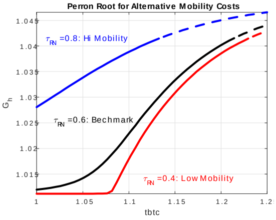
Human Capital



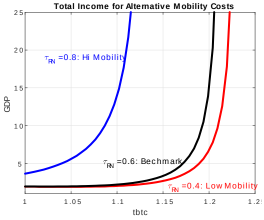
GDP

Mobility Costs

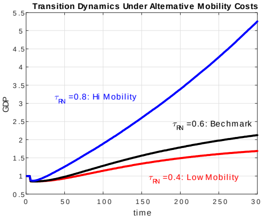
- **Benchmark:** $\tau_{RN} = 0.6$; **lower mobility costs** $\tau_{RN} = 0.8$, **higher mobility costs** $\tau_{RN} = 0.4$.



Perron Root $\mathcal{M}$



Cross BGP GDP



Transition dynamics

General Equilibrium

Production

- **Production:** Y_t final good; K_t: Structures; Q_t: bundle of tasks.

$$\begin{aligned}
\textbf{final output} \quad &: \quad Y_t = (K_t)^{\varphi} (Q_t)^{1-\varphi}, \\
\textbf{agg. bundle of tasks} \quad &: \quad Q_t = \left[\sum_{j=1}^{J} \left(Q_t^j\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \\
\textbf{bundle occupation j} \quad &: \quad Q_t^j = \left(\int_0^1 \left[q_t^j(x)\right]^{\frac{\eta-1}{\eta}} dx\right)^{\frac{\eta}{\eta-1}}, \\
\textbf{task } x \textbf{ occup. j} \quad &: \quad q_t^j(x) = z_t^{j,M}(x) M_t^j(x) + z_t^{j,H}(x) H_t^j(x).
\end{aligned}$$

- **Each Task x:** Workers j & machines: **perfect substitutes.**
  - $\left\{z_t^{M,j}(x),\ z_t^{H,j}(x)\right\}$ productivity levels, labor j & machines.
  - $\left\{H_t^j(x), M_t^j(x)\right\}$ total units total labor j & machines.

- **Productivities:** $\left\{z_t^{M,j}(x),\ z_t^{H,j}(x)\right\}$, **i.i.d. Frechet**: $\left\{\nu_{\mathbf{j}};\ A_t^{H,j}, A_t^{M,j}\right\}$.

Firm's Cost Minimization

- **Each Task x:**

$$c_t^j(x) = \min\left\{\frac{w_t^j}{z_t^{H,j}(x)}, \frac{r_t^M}{z_t^{M,j}(x)}\right\}.$$

- **Occupation Bundle of Tasks:**

$$C_t^j = \min_{q_t^j(x)} \int_0^1 c_t^j(x)\, q_t^j(x)\, dx \quad \text{s.t.} \quad \left(\int_0^1 \left[q_t^j(x)\right]^{\frac{\eta-1}{\eta}} dx\right)^{\frac{\eta}{\eta-1}} = 1.$$

- **Aggregate Bundle of Tasks:**

$$C_t^Q = \min_{\{Q_j\}_{j=1}^J} \left[\sum_{j=1}^J C_t^j Q^j\right] \quad \text{s.t.} \quad \left(\sum_{j=1}^J \left(Q^j\right)^{\frac{\rho-1}{\rho}}\right)^{\frac{\rho}{\rho-1}} = 1.$$

Firm's Cost Minimization: Closed-Form Solutions

- The probability that workers implement any task x in occupation j is:

$$\pi_t^{H,j} = \frac{\left(A_t^{H,j} / w_t^j\right)^{\nu}}{\left(A_t^{M,j} / r_t^M\right)^{\nu} + \left(A_t^{H,j} / w_t^j\right)^{\nu}}.$$

  Machines implement it with probability $\pi_t^{M,j} = 1 - \pi_t^{H,j}$.

- The minimized unitary costs C_t^j of bundle from each occupation j are:

$$C_t^j = \Gamma\left(1 + \frac{1-\eta}{\nu_j}\right)^{\frac{1}{1-\eta}} \left[\left(A_t^M / r_t^{M,j}\right)^{\nu_j} + \left(A_t^{H,j} / w_t^j\right)^{\nu_j}\right]^{\frac{-1}{\nu_j}}.$$

- The minimized unitary costs C_t^Q of the aggregate bundle of tasks is:

$$C_t^Q = \left\{\sum_{j=1}^{J} \Gamma\left(1 + \frac{1-\eta}{\nu_j}\right)^{\frac{1-\rho}{1-\eta}} \left[\left(A_t^{M,j} / r_t^M\right)^{\nu_j} + \left(A_t^{H,j} / w_t^j\right)^{\nu_j}\right]^{\frac{\rho-1}{\nu_j}}\right\}^{\frac{1}{1-\rho}}.$$

- The price of the final goods is

$$P_t = \left[\varphi^{-\varphi}(1-\varphi)^{\varphi-1}\right]\left(r_t^K\right)^{\varphi}\left(C_t^Q\right)^{1-\varphi}.$$

Aggregate Output and Factor Prices

Given $\left\{A_t^{M,j}, A_t^{H,j}\right\}$ and supplies $\left\{M_t^j, H_t^j, K_t\right\}$: the **equilibrium quantities** are

$$Q_t^j = \Gamma\left(1+\frac{1-\eta}{\nu_j}\right)^{\frac{1}{\eta-1}}\left[\left(A_t^{M,j}M_t^j\right)^{\frac{\nu_j}{1+\nu_j}}+\left(A_t^{H,j}H_t^j\right)^{\frac{\nu_j}{1+\nu_j}}\right]^{\frac{1+\nu_j}{\nu_j}},$$

$$Q_t = \left[\sum_{j=1}^{J}\Gamma\left(1+\frac{1-\eta}{\nu_j}\right)^{\frac{\rho-1}{\rho(\eta-1)}}\left[\left(A_t^{M,j}M_t^j\right)^{\frac{\nu_j}{1+\nu_j}}+\left(A_t^{H,j}H_t^j\right)^{\frac{\nu_j}{1+\nu_j}}\right]^{\frac{(1+\nu_j)(\rho-1)}{\nu_j\rho}}\right]^{\frac{\rho}{\rho-1}},$$

$$Y_t = \left(K_t\right)^{\varphi}\left[\sum_{j=1}^{J}\Gamma\left(1+\frac{1-\eta}{\nu_j}\right)^{\frac{\rho-1}{\rho(\eta-1)}}\left[\left(A_t^{M,j}M_t^j\right)^{\frac{\nu_j}{1+\nu_j}}+\left(A_t^{H,j}H_t^j\right)^{\frac{\nu_j}{1+\nu_j}}\right]^{\frac{(1+\nu_j)(\rho-1)}{\nu_j\rho}}\right]^{\frac{(1-\varphi)\rho}{\rho-1}}.$$

and **equilibrium prices** (real rental rates and wages) $\left\{\rho_t^K, \rho_t^{M,j}, \omega_t^j\right\}$ are:

$$\rho_t^K = \varphi\frac{Y_t}{K_t},$$

$$\rho^{M,j} = (1-\varphi)\,\Gamma\left(1+\frac{1-\eta}{\nu_j}\right)^{\frac{1}{\eta-1}}\left(\frac{A_t^{M,j}M_t^j}{Q_t^j}\right)^{\frac{\nu_j}{1+\nu_j}}\left(\frac{Q_t^j}{Q_t}\right)^{\frac{\rho-1}{\rho}-\frac{\nu_j}{1+\nu_j}}\frac{Y_t}{M_t^j},$$

$$\omega^j = (1-\varphi)\,\Gamma\left(1+\frac{1-\eta}{\nu_j}\right)^{\frac{1}{\eta-1}}\left(\frac{A_t^{M,j}H_t^j}{Q_t^j}\right)^{\frac{\nu_j}{1+\nu_j}}\left(\frac{Q_t^j}{Q_t}\right)^{\frac{\rho-1}{\rho}-\frac{\nu_j}{1+\nu_j}}\frac{Y_t}{H_t^j}.$$

Capital Owners

- **Preferences:**

$$U_t^K = \sum_{s=0}^{\infty} \beta^t \frac{\left(c_t^K\right)^{1-\gamma}}{1-\gamma},$$

- **Investment:** Curvature a la Lucas-Prescott:

$$\begin{aligned} K_{t+1} &= \left(1-\delta^K\right) K_t + \xi^K I_t^K, \text{ and} \\ M_{t+1}^j &= \left(1-\delta^M\right) M_t^j + \xi^M I_t^{M,j}, \; j = 1, 2, \ldots J \end{aligned}$$

- **Budget constraint:** for any period t:

$$\rho_t^M \sum_{j=1}^{J} M_t^j + \rho_t^K K_t + R_t B_t = c_t^K + I_t^K + \sum_{j=1}^{J} I_t^{M,j} + B_{t+1},$$

General Equilibrium: Existence of BGP

Theorem

Consider an economy that satisfies the parameter restrictions laid out above. Moreover, assume a constant, strictly positive machine productivity A^M and a workers productivity $\left\{A^j\right\}_{j=1}^J$, that grows at a constant rate $G_A \geq 1$ such that the BE of the workers are well defined. Then, there exists an equilibrium BGP.

Dynamic Hat Algebra

- If at initial period $t = 0$:
  - **Observed:** shares/transitions:
    - employment, human capital shares $\left\{\theta_0^{0,j}, H_0^{0,j}\right\}$,
    - transitions $\{\ \mu_{-1},\ \mathcal{M}_{-1}\ \}$,
    - current-to-total income Φ_0^j,
    - factor payments shares, $\left\{\pi_0^{H,j}, \pi_0^{M,j}\right\}$;
    - non-structures value added share of occupation j $\left\{\varrho_0^j\right\}$.
  - **Calibrated/estimated:** parameters $\{\beta, \gamma;\ \delta^M,\ \delta^K;\ \varphi, \rho, \nu_j;\ \alpha\ \}$.
- **Then:** the following can be written in changes relative to the BGP.
  - the sequential equilibrium for $t = 1, 2, \ldots$
  - responses to unanticipated changes, at $t = 0$, in $\left\{A_t^{H,j},\ A_t^{M,j},\ \xi_t^M\right\}$.
  - **not needed:** parameters (τ, λ), levels (A^H, A^M, ξ^K, ξ^M).

Quantitative Analysis

Quantitative Analysis

- **Model:** Add *Non-Pecuniary Costs* $\chi : J \times J \to \mathbb{R}$:

$$v(j,\epsilon) = \frac{\left(w^j\right)^{1-\gamma}}{1-\gamma} + \beta \max_{\ell} \left\{ \left(\tau_{j,\ell}\epsilon^{\ell}\right)^{1-\gamma} \left[\chi_{\mathbf{j},\ell} \cdot \mathbf{v}^{\ell}\right] \right\},$$

  to match switches in the data.

- **Initial BGP:** Late 1970s.

- **Capital Embedded Technology Advancement: 1980 - 2010.**
  - ξ_t^M: to match changes in in the price of equipment.
  - Perfect foresight.

Calibration

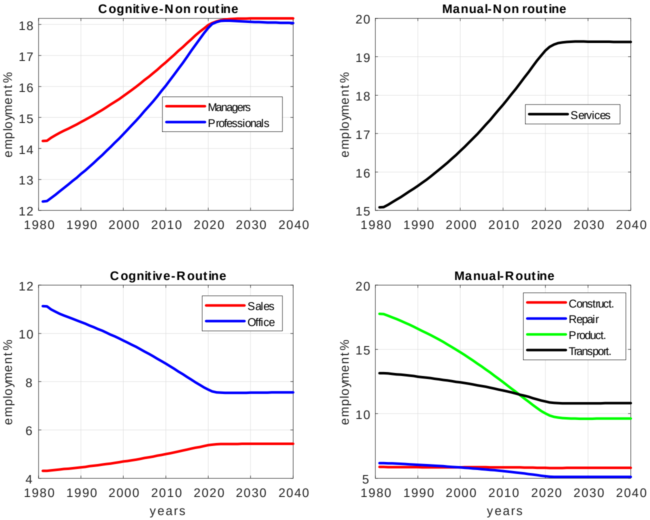
parameter / variable		value
time period: 1 year		
No. of occupations	J	9
yearly model, discount factor	β	0.95
labor market exit prob.		0.03
risk aversion	γ	2
shape Frechet workers	α	13
exogenous growth rate of labor prod.	$A_t^{H,j}$	1.01
employment share of new cohort	θ_0^0	data 1970s
human capital of new cohort	H_0^0	"
occupational mobility matrix	μ_{-1}	"
human capital transition matrix	$\mathcal{M}_{-1}$	"
employment share of all workers	θ_0	"
human capital of all workers	H_0	"
ratio of current vs permanent income	Φ_0^j	see Appendix B
depreciation rate machines	δ^M	0.125
depreciation rate structures	δ^K	0.05
share of structures	φ	0.13
elasticity between occupations	ρ	1.00
real rate–internatinally given	R/P	$\beta^{-1}(G_A)^\gamma$
share of occ j value added	ϱ_0^j	$(H_0^j \;/\; \sum_k H_0^k)$
share of labor in occ j value added	$\pi_0^{H,j}$	$0.76\ \forall j$

- Extension of CDP (2019): Solve for transition in changes

Calibration of Production Elasticities

Employment Changes	**Data**	**Model**	ν_j
Managers	3.1	3.9	0.08
Professionals	8.3	5.8	-0.90
Services	3.7	4.2	0.05
Sales	1.8	1.1	0.10
Office & Admin	-4.9	-3.6	2.00
Construction	-0.6	-0.8	1.00
Repair & Maint	-0.9	-1.1	1.50
Production	-9.1	-8.0	2.50
Transportation	-1.4	-2.3	1.50

Worker Transitions: Employment shares



Changes in Employment Composition

Table: Contribution of Responses of Old and New Cohorts.

Occupations	Initial share	Final share	Contribution of Changed Behavior New Cohorts Only	Old Cohorts Only
Managers	14.2	18.1	0.3	3.6
Professionals	12.3	18.0	0.7	5.0
Services	15.1	19.3	0.5	3.7
Sales	4.3	5.4	0.1	1.0
Office & Admin	11.1	7.6	-0.4	-3.2
Construction	5.9	5.8	0.0	-0.1
Repair & Maint	6.2	5.1	0.0	-1.0
Production	17.8	9.9	-1.0	-7.1
Transportation	13.2	10.9	-0.1	-2.1

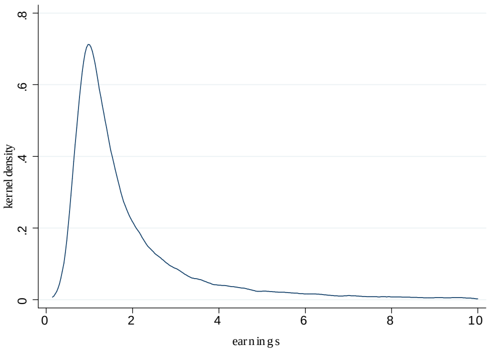
Counterfactuals

Parameters for Counterfactuals

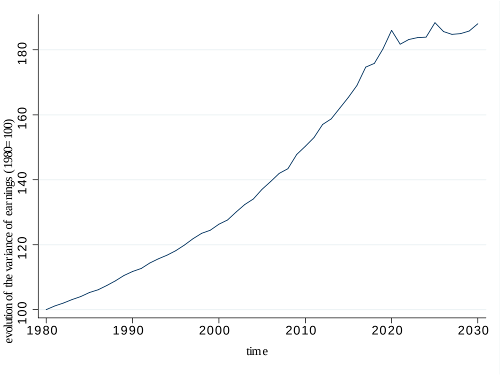
Table: Parameters affecting earning dynamics

Occupations	λ	τ^0	χ^0
Managers	0.953	1.043	1.000
Professionals	0.975	0.923	0.914
Services	0.963	0.579	1.675
Sales	0.952	0.806	1.145
Office & Admin	0.967	0.750	1.224
Construction	0.952	0.895	1.104
Repair & Maint	0.948	0.964	1.055
Production	0.952	0.933	1.241
Transportation	0.952	0.824	1.349

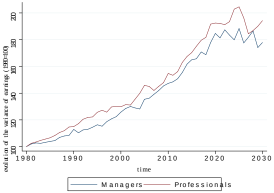
Initial Distribution



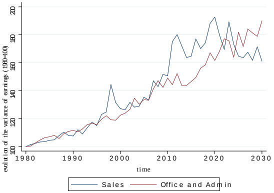
Evolution of Variance of Log-Earnings



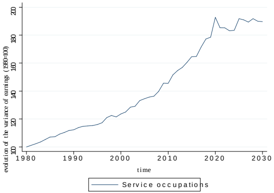
Variance of Log-Earnings, Occupations



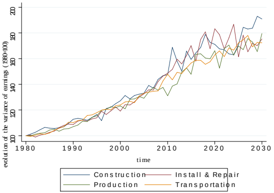
Cognitive, non-routine



Cognitive, Routine



Manual, non-routine



Manual, routine

Counterfactuals: Mobility Costs, Elasticities

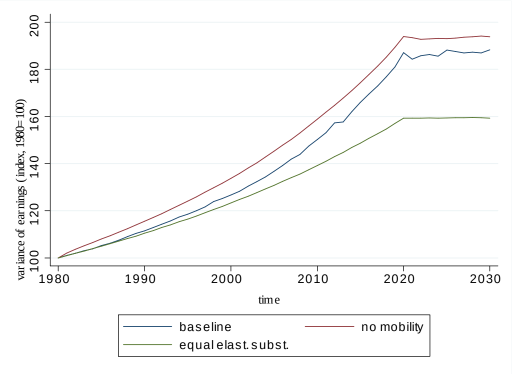


Figure: Inequality: Benchmark and Alternatives

Closing Remarks

- **A Dynamic Roy Model**
  - **Analytical characterization**
  - Individual Problems
  - Transition Functions:
    - Worker flows & Invariant Distributions.
    - Human Capital Aggregates & Limiting Eigenvectors.
  - A novel mechanism for endogeous growth.
- **Production: Assignment of Tasks to Workers or Machines.**
  - Characterization of Prices and Factor Shares.
  - Balanced Growth Paths (BGP).
- **Impact of Technological Change: Task-Biased Technical Change:**
  - **Method:** Dynamic Hat Algebra
  - **Quantitative Exercises:**
    - Burst of reallocation, growth and inequality.
    - Counterfactuals

Supporting Slides

Extensions and Variations

- **Alternative Timing:**
  - Occupation Choice at the beginning of the period.
- **Transient and Permanent Costs of Occupation Switches:**
  - Costs on current consumption or one period transit costs.
- **Heterogeneity and Age Dependence:**
  - Heterogeneous workers $e = 1, ..., E$. Heterogenous transfer matrices τ^e.
  - Stochastic Aging.
- **Endogenous On-the-Job Training (Ben-Porah)**
  - Investment i:
$$c_t = whe^{-\theta_0 i}, \quad h' = he^{\theta_1 i}.$$
- **Formal Education and Entry to Labor Markets:**
  - Choice of h_0 and matrix τ^{major}.
- **Deterministic Aggregate variations:**
  - Time varying wages $\{w_t\}_{t=0}^{\infty} \implies$ Time varying values $\{v_t\}_{t=0}^{\infty}$

Introduction: Standard Roy Models

for support

- **Workers' Problems:** Wages $\mathbf{w} \in \mathbb{R}^J_{++}$, skills $\epsilon \in \mathbb{R}^J_{+}$.
  - **Occupation Choice by Income Maximization:**

$$\max \left\{ w_1 \epsilon_1, w_2 \epsilon_2, ..., w_J \epsilon_J \right\}.$$

  - **Assignment by Comparative Advantage:**

$$j \text{ chosen } \textbf{iff} \quad \epsilon_j > \max \left\{ \frac{w_i}{w_j} \epsilon_i \right\}_{i \neq j}.$$

- **General Equilibrium:** $\xi \sim$ *Extreme Value* distributions (e.g. Frechet)
  - **Assignment Workers** type e to jobs j

$$\Pr(e, j) = f_e(w_j, w_{-j}).$$

  - **Aggregate Skill & Market Clearing Prices.**
- **Quantitative Macro Applications:** Many, recent e.g.:
  - **Matching of Tasks and Skills:** Costinot & Vogel (2015).
  - **Between Group Inequality:** Burstein et al.(2018).
  - **Aggregate Costs of gender/race discrimination:** Hsieh et. al (2018)
  - **Cross-Country differences:** Monge-Naranjo et al. (2018).

This Paper: A Dynamic Roy Model

Workers' Problems: **Values** $v \in R^J$. **Stochastic Process:** $\{\epsilon_t\}_{t=0}^{\infty}$.

- Recursive Formulation: Bellman Equations for **v**.
- **Frechet ξ, CRRA preferences:** **Analytical characterization:**
  - Simple recursion for normalized expected values.
- **Transition Functions:** Workers and Human Capital
  - **Workers:** Invariant Distributions.
  - **Human Capital:** Perron-Eigenvectors.

Production: **Assignment of Tasks: Workers or Machines.**

- Characterization of Prices and Factor Shares.

General Equilibrium **Existence and Characterization.**

- **Static Equilibrium:** Analytic Characterization.
- **Balanced Growth Paths:** Existence. Narrowing on uniqueness...
- **Transitional Dynamics:** Hat-algebra.

Impact of Technological Change: **Automation, Growth & Inequality**

Some Observations on Occupation Switches

Occupation mobility matrix (1994-2010)

	prob. of stay in same occup	cond. on switching, prob. to move to			
		NRC	RC	NRM	RM
Non-routine cognitive	0.51	0.44	0.36	0.05	0.14
Routine cognitive	0.52	0.47	0.19	0.13	0.22
Non-routine manual	0.45	0.21	0.26	0.19	0.33
Routine manual	0.55	0.17	0.20	0.17	0.46

Wage loss, occupation switching and occupational groups (1994-2010)

Dep. variable: change in log wage	Coeff	Robust std. error
Stay in same occupation	-0.047 ***	0.008
Different occupation		
same group	-0.106***	0.017
different group	-0.151 ***	0.013

Computed using CPS-DWS using workers that were displaced due to plant closing, insufficient work or position abolished in the past three years. Sample restricted to ages between 25 and 65 with full-time jobs at the time of displacement and with a full-time job at the time of the survey. Occupation defined at 2-digit SOC. An occupation switch is defined as a different 2-digit SOC occupation. NRC = Non-routine cognitive, RC = Routine cognitive, NRM = Non-routine manual, RM = Routine manual.

Illustration: Dynamic vs Static Occupation Choice

Stagnant, Flexible and Growth Occupations

- Three Jobs: $J = 3$. $\lambda = [1, 1, 1]' / \Gamma(1 - 1/\alpha)$.
  - **Jobs 1:** High pay, low growth, low flexibility.
  - **Jobs 2:** Low pay, low growth, high flexibility.
  - **Jobs 3:** Low pay, high growth, low flexibility.

$$\textbf{Jobs}: \ \mathbf{w} = \begin{bmatrix} 1.75 \\ 1 \\ 1 \end{bmatrix}; \qquad \boldsymbol{\tau} = \tau[j, \ell] = \begin{bmatrix} \mathbf{1.00} & \mathbf{0.75} & \mathbf{0.5} \\ \mathbf{0.95} & \mathbf{1.00} & \mathbf{0.95} \\ \mathbf{0.5} & \mathbf{0.75} & \mathbf{1.075} \end{bmatrix}$$

Discounting: Patient vs. Impatient Occupation Choices

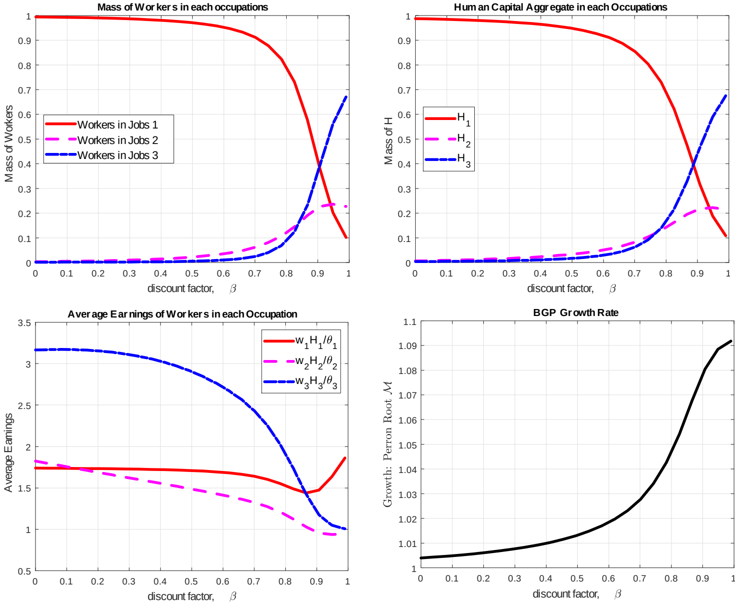
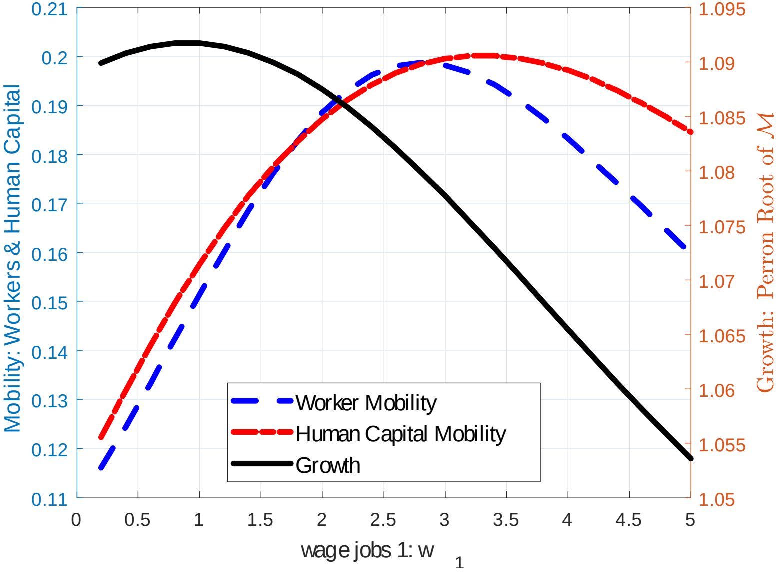


Illustration: The Perils of High-Paying/Stagnant Jobs



General Equilibrium

Definition

Given an initial population of workers and their human capital, $\left\{\theta_0^j,\ H_0^j\right\}_{j=1}^J$, initial stocks of machines and other physical capital $\{M_0, K_0\}$, and exogenous sequences $\left\{A_t^j, A_t^m\right\}_{t=0}^\infty$ an equilibrium is **(i)** a *price system* $\left\{w_t^j, P_t, r_t^K, r_t^M, R_t\right\}_{t=0}^\infty$, **(ii)** individual worker occupation decisions $\left\{v_t^j, \mu_t\right\}_{t=0}^\infty$, **(iii)** individual firm tasks-allocation choices $\left\{\pi_t^j, \pi_t^M\right\}_{t=0}^\infty$, **(iv)** aggregate *vectors of workers and human capital across occupations, stocks of machines and other physical capital,* $\{\theta_t, H_t, M_t, K_t\}_{t=0}^\infty$, and, **(v)** aggregate output, *worker and human capital reallocations, and flows of investments and of consumption of the owners of capital,* $\{Y_t,\ \mu_t,\ \mathcal{M}_t, I_t^K, I_t^M, c_t^K\}_{t=0}^\infty$ such that: **(a)** Given $\left\{w_t^j, P_t, r_t^K, r_t^M\right\}_{t=0}^\infty$, $\left\{v_t^j, \mu\right\}_{tt=0}^\infty$ solve the the workers lifetime optimization , $\left\{\pi_t^j, \pi_t^M, P_t\right\}_{t=0}^\infty$ solve the firms optimization; and capital owners invest optimally, i.e. according to their Euler equations. **(b)** factor markets clear hold for every t, and **(c)** the population of workers and human capital allocation evolve according to individual choices.